International Journal of Human Capital in Urban Management
(IJHCUM)

Homepage: http://www.ijhcum.net/

ORIGINAL RESEARCH PAPER

The role of proportional systems in the forming of Sassanid square domes

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ARTICLE INFO

Article History:
Received 20 December 2017
Revised 12 Fabruary 2018
Accepted 5 March 2018

Keywords:
Building Techniques
Corners
Geometric and Mathematical Proportions
Sassanid Architecture
Square Domes

ABSTRACT

One of the common monuments in the Sassanid era was the square dome. Interestingly, corners and parts of the square dome were always geometrically proportionate to each other. Square domes have been mainly studied in terms of their historical background as well as the materials used. The present research, however, aimed to examine the geometric proportions governing the formation of square domes in the Sassanid era. The data were mainly gathered from field studies. First, the intended monuments were individually studied: the sizes were carefully measured and recorded. Then, the data were simulated by Auto CAD and the geometric repetitive proportions of the parts were extracted. The results indicated that the ratio of the corner depth to the radius of the dome circle was 0.3; the ratio of the angle length to the inward length was 0.11 and that of the outward side to the inward side was 1.6

DOI:10.22034/IJHCUM.2018.02.01

INTRODUCTION

As a fundamental principle in the traditional art of Persian building, the functioning and stability of a building follow its geometry. A perfect geometry guarantees stability. This principle can be traced in many historical Persian buildings. Meanwhile, during the Sassanid Empire, the most essential innovations occurred in dome architecture with the construction of square domes, which were often erected on reception halls of palaces (Smith, 1971). These domes were placed on the first samples of squinches, which had a lasting effect on dome development in Islamic architecture (Grabar, 1963; Creswell and Allan., 1990). Meanwhile, geometry played a fundamental role in the design of these architectural monuments where the plan and elevation were set out in a framework of squares and equilateral triangles. The intersections of these gave all the important fixed points, such as the width and height of doors, the width, length and height of galleries, the position of inscriptions, etc. (Rafiei *et al*., 2005). Thus, the size of every part was related to every other part in some defined proportion. A building was not, therefore, a collection of odd components, but a harmonious configuration of proportionally related elements, which gave movement to space and satisfied the eye. It is through the system of proportions that all parts are harmoniously interrelated with and within the whole; therefore providing a pleasing and functional design. Some systems of proportions were based on the musical intervals, the human body, and the Golden Ratio. In another words, the right geometry made the building behave correctly (Hejazi, 2005).

As shown by prior studies, in the Sassanid period artisans and architects demonstrated

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high achievements in complex geometrical rules (sarhangi, 1999; Hejazi, 2005; Kharazmi *et al*., 2012; valibeig and mohammadi, 2016). This article aims to show the importance of geometry, especially proportional systems, as an influential component in the design process of Chahartaqi buildings. The application of practical geometry throughout these Sassanid buildings will be discussed and their geometrical ratios will be shown in order to arrive at a deep understanding of the connection between mathematics and art in ancient Persia. So far, the studies performed on square domes have focused mainly on the historical background as well as structural features. This study, however, aimed to examine them to achieve the proportional systems used in the buildings. The results obtained from such studies may help to have a better understanding of construction techniques in the past. In addition, recognition of architecturally dominant structures in a given era may offer new geometric perspectives to be used in today's architectural patterns. To achieve this, the current study aims to answer the following main questions:

1) Are there any specific geometric repetitive proportions between the outward length and the inward length in square domes?

2) What are the specific geometric repetitive proportions in the plan view and elevation of the square domes?

3) Are there any similar proportions between the corner depth and the radius of the dome circle?

After careful examination of square domes in terms of dimensions of different parts (in both plan view and elevation), the existing geometric proportions were extracted through computerized modeling. Existing research in the field of practical geometry relevant to architectural buildings mostly pertains to geometric patterns of Islamic architecture (Ozdural, 2000; Bonner, 2003). Scant research exists exploring the same subject in ancient Persian architecture and proportions (Kharazmi *et al*, 2012). There are some comprehensive research works dedicated to the field of metaphysical (Ardalan and L, 1973) and mathematical (Kritchlow, 1969; Hejazi, 2005) aspects of Sassanid architecture that have made it possible to uncover some of the profound knowledge used in Persian traditional architecture. Geometrical analysis of many Persian historical buildings has proven that a complete knowledge of proportions, in particular, the Golden Ratio was widely used in Persian architecture and it was the basis of Persian aesthetics (Hejazi, 2005). Kharazmi *et al.,* (2012) conducted a research to examine whether or not Sassanid stucco followed specific geometrical rules. By analyzing Sassanid stucco ornaments, they show the knowledge of geometry and mathematics possessed by ancient Persians. The seven frieze groups and seventeen wallpaper groups have been used to show the application of repeating units and symmetry. They provide a better explanation for the creation of the Sassanid stucco patterns, and thus a better understanding of the ornaments. Hejazi (2005) conducted research to analyze geometrical characters of Persian traditional buildings. He proved that there are many proportions that are used in these buildings and that these geometrical relations provide the basis for the whole building. Put another way, the design of plans, elevations, geometric and architectural patterns, and mechanical and structural features, can be proved through geometrical analysis of Persian historical buildings. He showed that systems of proportions in architecture originated from musical ratios, sacred geometry in nature or mathematical patterns. Using geometrical analysis through the Persian architecture, he found the similar ratios in Taj-al-Mulk's dome in Isfahan based which he defines the Golden Ratio that had been masterly used over the design of the building. Similarly, there is the same ratio in the elevation of the dome representing this module. Mehdizadeh *et al.,* (2011) explored the mathematical and physical ratios used in building dimensions. They showed that these proportions were used in the construction process to make the building perform correctly. Through this research, a new system of proportion, called "Hanjar triangle", was introduced. Their results showed that the key element to the appropriate geometry is the modular system, where proportions and relations emerge. They concluded that all these dimensions were derived from edges of the right-angled triangle. Certain investigations, which examined Sassanid art in Firouzabad and Bishapur, made it clear that vaults and domes were structurally and functionally important elements (Ghrishman *et al*., 1971). Likewise, during the restoration process of Dokhtar Castle in 1973, it was revealed that there were certain structural similarities and differences between the fire temples (Hugi, 1977). Persian architects have made the most of geometric proportions, both before and after the Islamic era, in their works (Hejazi, 2005). Among the geometric proportions in Persian square domes, several cases have been mentioned including the ratio of column width to lateral side length which ranged between 1, 3/2 and 3/9. The ratio creates a 23.5° arc distance between the column corners and the resultant line of sight. Interestingly the 23.5° arc distance equals the solar inclination angle (Vanden Berghe, 1977). Several researchers have also studied how the geometric features of the corners have undergone changes. However, whether there is a geometric proportion between the architectural elements of the square domes has not been studied (Godard, 1965; Hejazi and Saradj, 2014). Exploring Persian traditional buildings, Godard (1965) found a quantitative relation in the Niasar Chahar Taghi, which has a proportion of 1/2 in the relation of column's width to the distance between two columns. Kovskaia and Golombek (1985) ferret out geometrical proportions around the fourteenth century in central Asia. They introduced a new proportion called GAZ, which utilized as a module throughout the buildings. Namjou (2014) conducted a study on Chahar Taghi Buildings over the Sassanid era in Iran. He perceived all the buildings through two categories of plan and elevation based on their geographical location divided into four geometrical groups. The measured attributes include the dimensions of plan, columns and height of the building. According to this, the smallest Chahar Taghi is Mahneshan with 16 square meter however; the biggest one has 625 square meter belonged to the Ghasr-e-Shirin. Comparing the results, it can be seen some repetitive ratios over the dimensions of 100 measured buildings; however, this study does not develop any quantitative framework to identify these proportions.

MATERIALS AND METHODS

The Sassanid era is one of the historical periods in which square domes were very common. (Namjoo, 2014) They were especially used as fire temples. Fortunately, a considerable number of famous square domes have survived (Fig. 1).

Since it is not possible to examine all the surviving

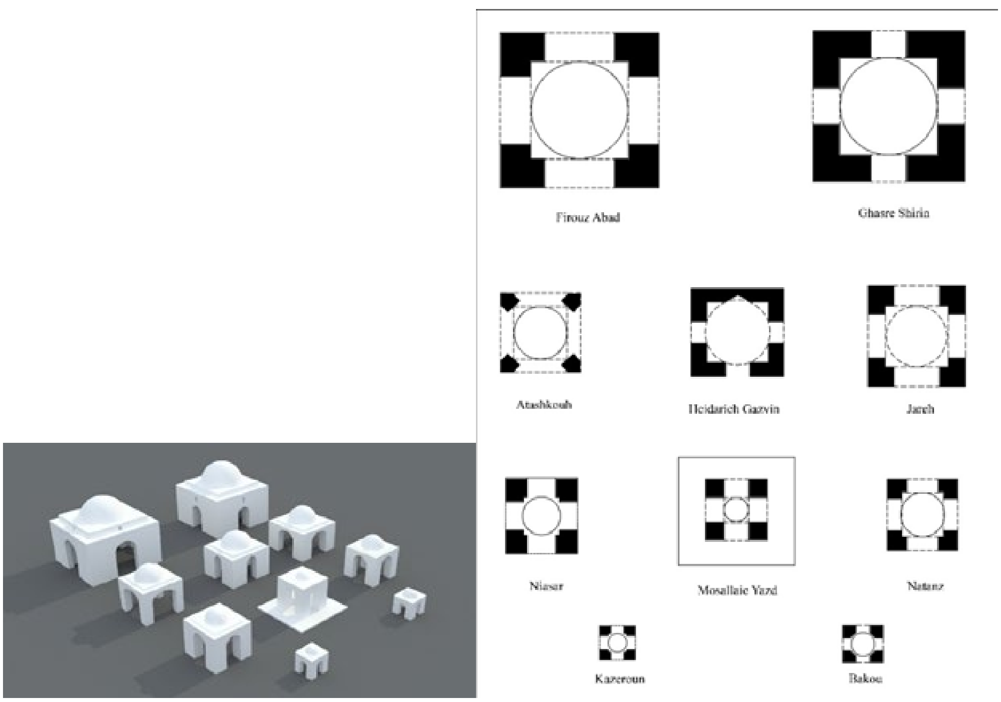


Fig. 1. Explored square domes in Iran

square domes in this study, only square domes limited to the Sassanid era and to the country of Iran were examined. In addition to geometric characteristics, other selection criteria for the square domes included geographic factors, being inside or outside of the city, as well as the dimensions. In addition, only typical cases were selected to represent a large group of square domes. The selection of similar domes was avoided (Fig. 2). On this basis, a total of twenty Sassanid square domes were selected and studied in this research including fire temples in Izadkhvast, Jareh, Kheyrabad, Natanz, Khorramdasht, Neyasar, Bazeh Hour, Kazerun, Zahr shir, Sarvestan, Borzou, Konar siah, Firuzabad, Takhtkaykavous, Chahar taghi Nevis, Karateh, Neghar, Atashkadeh Azar goshtasb, Ghasreshirin, and Minoodasht.

After collecting the needed data using library sources, and mainly, field studies, the dimensions of the square domes were measured and recorded. Since the most commonly used tool to analyze monuments is computerized modeling (Valibeig *et al*., 2012), the dimensions of all parts (both in plan view and in elevation) were modeled, analyzed and examined. The results indicated that there were considerable similarities between different parts of the square domes studied. The results obtained are supported by the outcomes of previous research on similar structures (Godard, 1965).

RESULTS

In architecture, a square dome, as the name implies, refers to a square-base structure with a dome overhead. Square domes are among the important forms in Sassanid architecture and have played a key role in its evolution. The main constituents of a square dome include four thick piers each of which are located in one of the four corners; four arches each of which connects the four piers to each other; a dome built over the arches with the help of four square arches, and four archways formed on four sides of the whole structure (Zamani, 1977). In square domes, there are specific geometric proportions. In order to

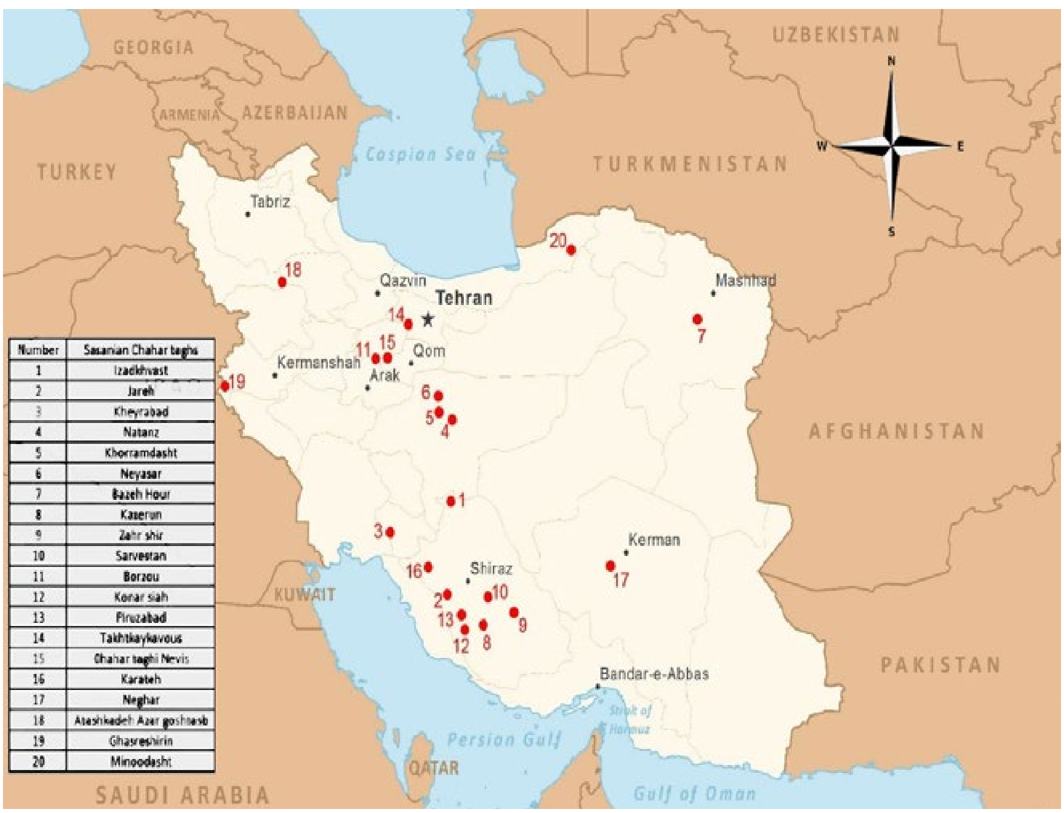


Fig. 2. Geographical locations of the square domes studied in Iran- Scale: 1/10000000

Table 1. Specifications in a square dome plan designated by conventional symbols

Conventional Symbols in the plan		Conventional Symbols in the plan	
a	outward length	m	Radius from the dome center to the centers
b	inward length	n	Corner spaces
c	length of the angular pier	p	Pier radius considered for forming corners
d	Inward thickness of piers	r	Dome radius

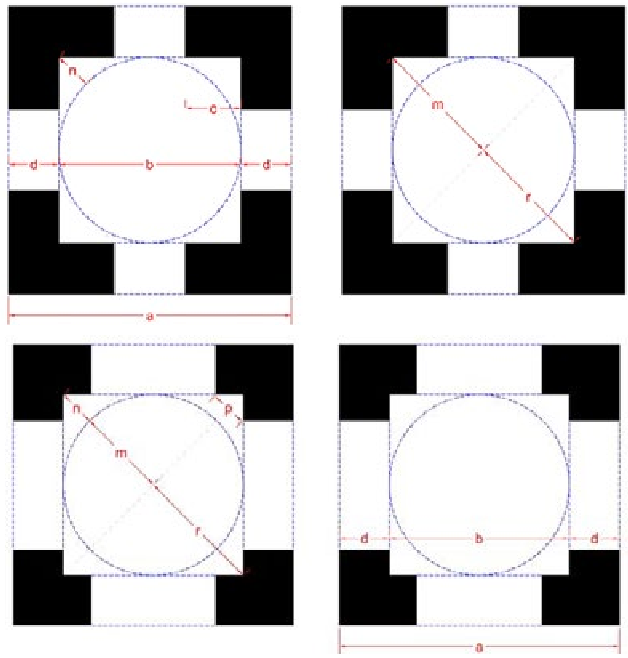
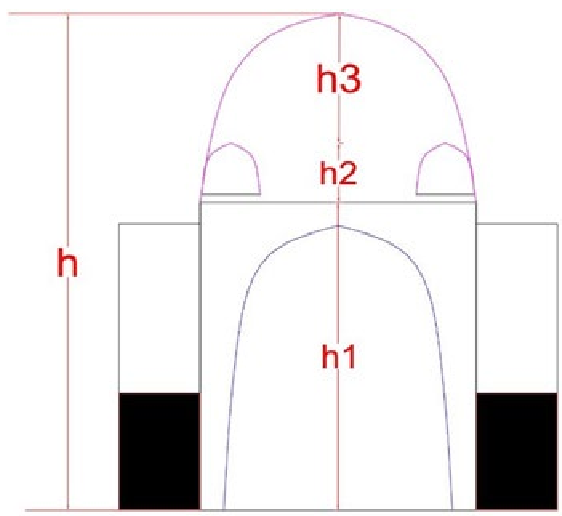


Table 2. Specifications in a square dome elevation designated by conventional symbols

Conventional Symbols on the view	
h	Structure total height
h_1	Height between the floor and the eaves
h_2	Height between the eaves and the head of openings
h_3	Height between the head of openings and dome ceiling



have a better understanding of these proportions, the dimensions under discussion in the plan, as well as in the elevation, are given in Tables 1 and 2, respectively.

The ratios a/b, c/b, c/a, p/n, p/m, n/r in the plan, and the ratios h2/h1, h2/h3, h3/h1+h2 in the elevation have been calculated and given in Tables 3, 4 and 5.

The examination of geometric repetitive proportions in Sassanid square domes supports the existence of a numerical constant/relationship. Based on the information given in Tables 1 and 4, However these dimension vary throughout the buildings (Namjoo, 2014), there is a few repetitive proportions among them. The ratios in the plan are as follows:

- The ratio of the length of the outward side to that of the inward side (relation 1-1).
- The ratio of the length of the angular pier to that of

Table 3. The results from examination of promotions in square domes

Row	Sasanian Chahar taghs	Geometric Proportions in Chahar taghi plans							Available Sizes in Chahar taghi plans						Geometric Proportions in Chahar taghi views		
		a	b	c	m	n	p	r	a/b	c/b	c/a	p/n	p/m	n/r	h_2/h_1	h_2/h_3	h_3/h_1+h_2
1	Izadkhvast	9.00	5.40	0.60	2.50	1.20	0.40	3.80	1.60	0.11	0.06	0.33	0.16	0.30	-	-	-
2	Jareh	11.80	7.20	0.80	3.50	1.50	0.50	5.00	1.60	0.11	0.06	0.33	0.14	0.30	-	-	-
3	Kheyrabad	7.50	4.40	0.40	2.20	1.00	0.40	3.20	1.70	0.09	0.05	0.40	0.18	0.31	-	-	-
4	Natanz	11.35	7.05	0.70	3.52	1.46	0.48	5.00	1.60	0.10	0.06	0.33	0.14	0.30	-	-	-
5	Khorramdasht	11.00	6.20	0.90	3.10	1.30	0.63	4.40	1.70	0.14	0.08	0.48	0.20	0.30	-	-	-
6	Neyasar	6.00	2.30	0.30	1.50	0.60	0.40	2.10	2.50	0.13	0.05	0.55	0.26	0.30	0.20	0.41	0.41
7	Bazeh Hour	13.20	6.00	1.40	3.00	1.30	1.00	4.20	2.20	0.23	0.11	0.70	0.33	0.30	-	-	-
8	Kazerun	7.00	4.30	0.50	2.10	0.90	0.40	3.00	1.60	0.11	0.07	0.44	0.19	0.30	-	-	-
9	Zahr shir	4.40	2.70	0.30	-	-	0.20	1.40	1.60	0.11	0.05	-	-	-	0.20	0.25	0.60
10	Sarvestan	4.70	3.40	0.90	1.60	0.70	0.60	2.30	1.40	0.26	0.19	0.85	0.37	0.30	-	-	-
11	Borzou	11.80	6.70	0.80	3.10	1.50	0.50	4.60	1.80	0.11	0.06	0.33	0.16	0.32	-	-	-
12	Konar siah	1.35	0.85	0.05	0.40	0.20	0.10	0.60	1.60	0.05	0.04	0.50	0.25	0.30	-	-	-
13	Firuzabad	8.50	5.20	0.80	2.60	1.10	0.50	3.70	1.60	0.15	0.09	0.45	0.19	0.30	-	-	-
14	Takhtkaykavous	5.20	3.50	0.30	-	-	-	-	1.50	0.08	0.06	-	-	-	0.33	0.25	1.00
15	Chahar taghi Nevis	8.95	5.05	0.55	2.50	1.05	0.35	3.50	1.80	0.11	0.06	0.33	0.14	0.30	-	-	-
16	Karateh	3.40	2.60	0.50	-	-	0.10	1.40	1.20	0.18	0.15	-	-	-	0.29	0.45	0.50
17	Neghar	7.80	4.60	0.75	2.30	0.95	0.53	3.25	1.60	0.16	0.09	0.55	0.23	0.30	-	-	-
18	Atashkadeh Azar goshtasb	3.30	1.60	0.40	-	-	0.30	1.15	2.10	0.25	0.12	-	-	-	0.32	0.31	0.78
19	Ghasreshirin	4.30	2.80	0.80	1.40	0.60	-	2.00	1.50	0.28	0.18	-	-	0.30	0.37	0.25	1.00
20	Minoodasht	8.80	5.00	0.60	2.80	1.20	0.40	4.00	1.70	0.12	0.06	0.33	0.14	0.30	-	-	-

Table 4. Geometric repetitive proportions in the plans of Sassanid square domes

$\frac{a}{b} = 1.6\ (1-1),$	$\frac{c}{b} = 0.11\ (1-2),$	$\frac{c}{a} = 0.05(1-3)\,,$
$\frac{p}{n} = 0.33(1-4),$	$\frac{p}{m} = 1.4 - 1.6(1-5),$	$\frac{n}{r} = 0.3(1-6)$

Table 5. Geometric repetitive proportions in the elevations of Sassanid square domes

$\frac{h_2}{h_1} = 0.2 - 0.4\ (2-1)$	$\frac{h_2}{h_3} = 0.25 - 0.45\ (2-2)$	$\frac{h_3}{h_1 + h_2} = 0.4 - 1.00(2-3)$

the inward side (relation 1-2).

- The ratio of the length of the angular pier to that of the outward side (relation 1-3).
- The ratio of the radius of the pier spent on the corner to that of the corner itself (relation 1-4).
- The ratio of the radius of the pier spent on the corner to the distance between the dome center and corner center (relation 1-5).
- The ratio of the distance between the corners to the circle radius (relation 1-6).

In addition, there are geometric repetitive proportions in the elevation (Table 2) as follows:

- The ratio of the height from the eaves to the opening head, to the height from the floor to the eaves (relation 2-1).
- The ratio of the height from the eaves to the opening head, to the height from the opening head to the dome ceiling (relation 2-2).
- The ratio of the height from the opening head to the dome ceiling, to the height from the floor to the eaves (relation 2-3).

CONCLUSION

The results indicate that geometric proportions, both before and after the Islamic era, have played an important role in Persian architecture. According to the examination of Sassanid square domes, the ratio 1.6 exists between the length of the outward side and that of the inward side (relation 1-1). Similarly, the numerical constant 0.3 exists for the distance between the corners and the circle radius (relation 1-6). In short, there exist geometric repetitive proportions among the architectural square domes (relations 1 and 2). The findings clearly attest to the fact that their architects have applied geometric proportions meticulously in the structure of Sassanid square domes. In general, Persian architecture is noteworthy in terms of geometric proportions and structural modules. Further studies on such modules may offer new approaches to be used in modern architecture.

CONFLICT OF INTREST

The author declares that there is no conflict of interests regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancy have been completely observed by the authors.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers for their comments on the paper.

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HOW TO CITE THIS ARTICLE

Abaeian, H.; Hashemi, M.; Valibeig, N.; Mohammadpour, S., (2018). The role of proportional systems in the forming of Sassanid square domes. Int. J. Hum. Capital Urban Manage., 3(2): 81-88.

DOI: 10.22034/IJHCUM.2018.02.01

url: http://www.ijhcum.net/article_32542.html